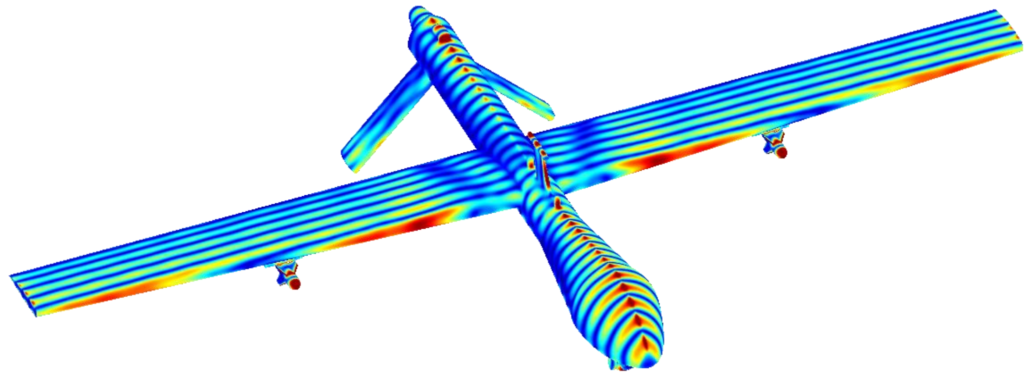
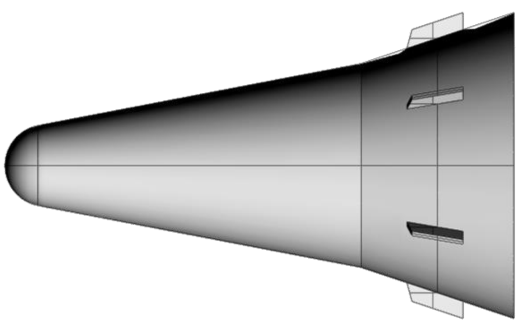
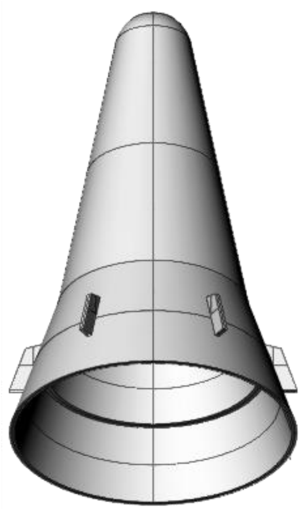
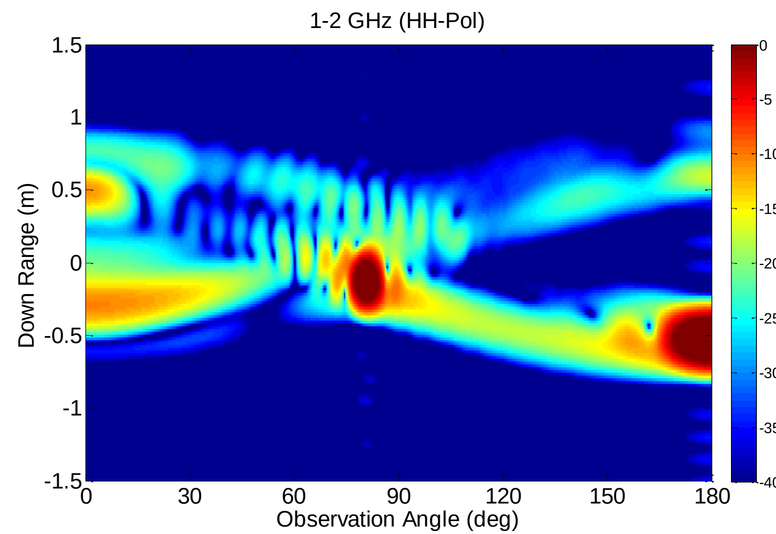
Adaptive Cross ApproximationSolver in the *Serenity* Radar Cross Section Prediction Code

Tripoint Industries, Inc.
September 1, 2020









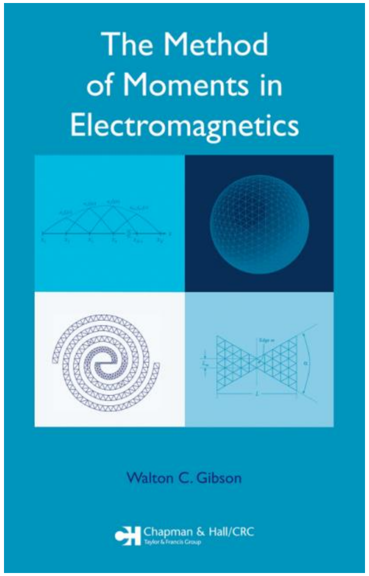
Serenity Overview

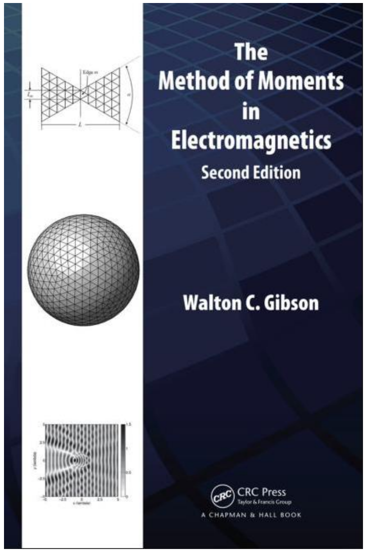
- ***Serenity*** **is a high-performance, full-wave radar cross section (RCS) solver employing the Method of Moments (MoM)**
  - **Solves the Surface Integral Equations (SIE) of scattering**
    - **Bounding surfaces of conductors and dielectric interfaces are meshed.**
  - **Objects having conducting (PEC) and bulk dielectric parts with junctions are fully supported. Dielectrics can be lossy.**
    - **Serenity enforces the EFIE on open conductors, CFIE on closed conductors, and PMCHWT on dielectrics.**
  - **Direct Block-LU decomposition of matrix in compressed form**
    - **Single-Level ACA and Multi-Level ACA (MLACA) Algorithms Supported.**
  - **Shared Memory Version**
    - **Parallelized via Threads, BLAS Level 3 operations accelerated via Intel Math Kernel Library (MKL) (CPU version), or NVIDIA cuBLAS (GPU version).**
  - **Distributed Memory (MPI) Version**
    - **Hybrid Approach, Parallelized via Threads on separate processes. Intel MKL used for all BLAS Level 3 operations.**
  - **Linux and Windows versions of *Serenity* are available.**

Serenity Overview (2)

- Developer: Walton C. Gibson
  - Industry expert in the Method of Moments applied to Electromagnetic Problems
  - Author of ***The Method of Moments in Electromagnetics***, 1st and 2nd Edition

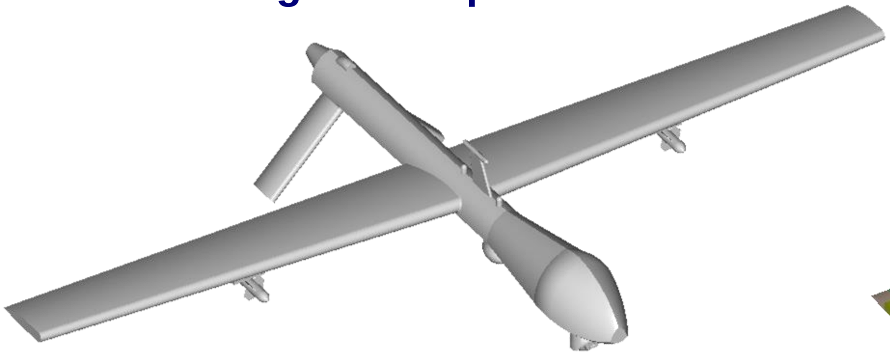
- *Serenity* Version History
  - v. 1.0 (2003): Written in C, supporting conducting objects and full matrix approach.
  - v. 2.0 (2012): Complete re-write in C++, full support for conductors and dielectrics.
  - v. 2.1 (2015): Added high-performance CPU and GPU-accelerated Adaptive Cross Approximation (ACA) solvers.
  - v. 2.2 (2019): Added high-performance CPU and GPU-accelerated Multi-Level Adaptive Cross Approximation (MLACA) solvers.

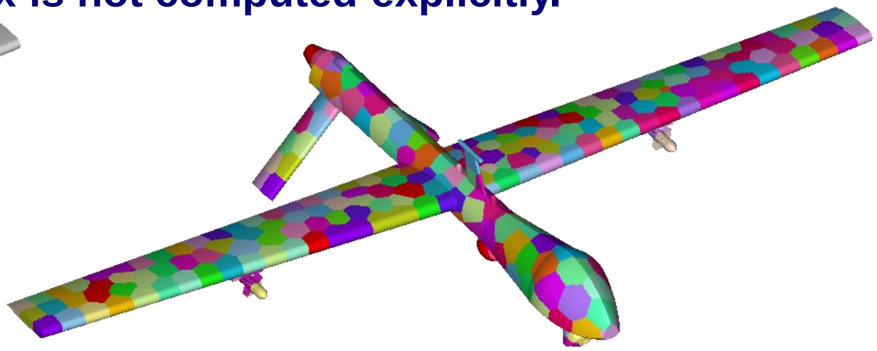




Single-Level ACA Solver Overview

- **As the MoM system matrix is dense, a full matrix approach is not tractable for electrically large problems**
  - **In-core solutions are limited to a few tens of thousands of unknowns**
  - **on current 2020 server-class systems.**
- **The *Serenity* Single-Level ACA solver groups basis functions into spatially local groups, breaking up the system matrix into block form.**
  - **Diagonal blocks due to interactions within a group are computed and stored as usual.**
  - **Rank-deficient off-diagonal blocks are computed and stored in compressed outer-product form on the fly using the ACA algorithm.**
    - **The ACA algorithm uses only selected rows and columns of each block – a significant portion of the matrix is not computed explicitly.**





Single-Level ACA Solver Overview (2)

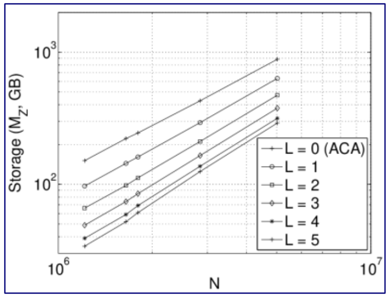
- **Compression of 98% or more versus the full matrix approach are possible on electrically larger problems.**
  - **Consider N = 500000, in single precision complex:**
    - **Full matrix: 1862 GB RAM**
    - **ACA with 98% compression: 37 GB RAM (solvable in-core!)**
  - **Memory Complexity is approximately $O(N^{4/3}\log N)$**
- **Block matrix is LU-factored in compressed form directly**
  - **LU factors also compressible via ACA**
  - **Can be done very fast using accelerated BLAS Level 3 functions**
    - **Intel MKL BLAS (SSE/AVX optimized for Intel processors)**
    - **NVIDIA CUDA BLAS (cuBLAS) on NVIDIA GPUs**
  - **Direct solve via LU decomposition eliminates problems caused in iterative solvers due to poorly conditioned system matrix**
- **The thousands of right-hand sides in a scattering problem can be solved simultaneously and efficiently**
  - **Right-hand side matrix is compressible in block form via ACA**
  - **Solution (current) matrix also compressible in block form via ACA**
  - **Nyquist sampled far fields can be computed *very* quickly**

Multi-Level ACA Solver Overview

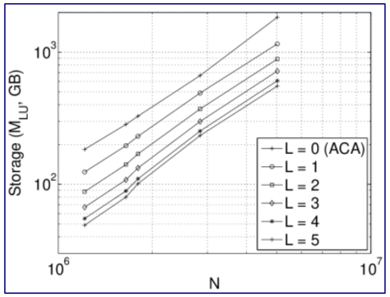
- **In the Multi-Level ACA (MLACA), Single-Level ACA groups are made larger and hierarchically subdivided into smaller, spatially local groups via binary-tree.**
- **Block matrix is again LU-factored in compressed form directly**
- **Butterfly, multi-level compression [1] applied to matrix blocks.**
    - **Diagonal *and* off-diagonal blocks are compressed in MLACA.**
- **MLACA yields much better compression than Single-Level ACA.**
    - **Memory Complexity remains approximately O($N^{4/3}$logN)**
    - **Extra compression is at the expense of more run time**
        - **Run-time increases are typically modest.**
    - **Accuracy is not adversely impacted.**

f_0	N	ACA		$L=1$		$L=2$		$L=3$		$L=4$		$L=5$	
		M_Z	M_{LU}	M_Z	M_{LU}	M_Z	M_{LU}	M_Z	M_{LU}	M_Z	M_{LU}	M_Z	M_{LU}
12.0	1213896	151	184	97	124	66	88	49	67	39	55	34	49
14.0	1645768	222	284	144	196	98	141	74	108	59	89	52	80
16.0	1799000	245	328	161	231	112	170	85	133	69	110	61	101
20.0	2843288	429	666	293	491	211	372	165	299	137	253	125	234
24.0	5022318	884	1841	634	1156	473	888	377	720	316	608	291	554

Example: Storage (M_Z and M_{LU} in GB) Versus # of Unknowns *N* for a Reentry Vehicle With Dielectric Nose, Comparing Single-Level ACA (L=0) to MLACA for Levels L= 1 to 5.



M_Z Versus *N*



M_{LU} Versus *N*

[1] Gibson, W. C. "Efficient Solution of Electromagnetic Scattering Problems Using Multilevel Adaptive Cross Approximation (MLACA) and LU Factorization," *IEEE Trans. Antennas Propagat.*, vol. 68, pp 3815-3823, May 2020.

CPU vs. GPU-accelerated ACA Solver

Tripoint Industries, Inc.

- **Comparison executed on a Dell Precision T7900 Workstation**
  - **Dual, 12-core Intel Xeon CPUs (E5-2690 v3), 256 GB RAM**
- **CPU ACA solver uses Intel Math Kernel Library MKL, 2019 Version**
  - **All Operations Performed on CPU**
- **GPU ACA Solver uses NVIDIA CUDA 10.0 and 2 GTX 1080 Ti GPUs**
  - **Single and Multi-GPU configurations (pictured) are supported by *Serenity***
  - **Matrix Filling and Factorization Performed on GPU, Compression on CPU**
- **GeForce GTX 1080 Ti released in March, 2017**
  - **Pascal Architecture, 3584 CUDA Cores, 12 GB of GPU Memory**
  - **~10 TFlops (single precision)**

EVA GeForce GTX 1080 Ti GPU

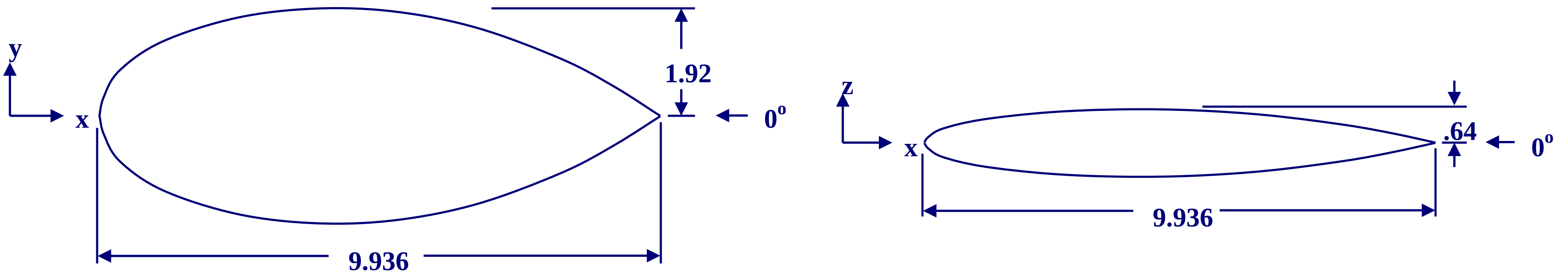
Multi-GPU (x4) Configuration

CPU vs. GPU-accelerated ACA Solver (2)

- Compare matrix fill and factor times of PEC spheres of increasing size.
- 24 CPU threads are used.
- Using GPUs, matrix fill is about ~3 times faster than using the CPU.
- Using GPUs, LU factorization is ~5-7 times faster than using the CPU.
  - Speed increase of the GPUs grows with problem size.
- Adding additional GPUs will improve factorization time significantly
  - If the user adds additional GPUs, *Serenity* can utilize them right away

Test Case	*N*	M_Z (GB)	T_Z (CPU)	T_Z (2x GPU)	M_{LU} (GB)	T_{LU} (CPU)	T_{LU} (2x GPU)
4λ PEC Sphere	122880	6	53	19 (2.8x)	7	213	41 (5.2x)
8λ PEC Sphere	491520	30	291	90 (3.2x)	37	3935	562 (7.0x)
16λ PEC Sphere	1966080	174	2211	634 (3.4x)	248	133297	21485 (6.2x)

Example: UT Austin PEC Almond (1/3)



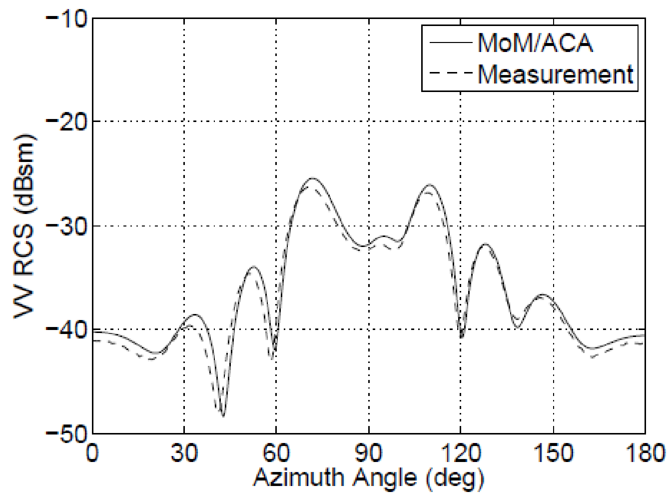
- Exterior dimensions identical to NASA Almond from the EMCC benchmarks[1]
- *Serenity* uses a single-level ACA
  - Target ACA group size: 2500
- UT Austin test article 3D printed from resin and coated with conducting paint
- RCS computed and compared at 2.58, 5.125, 7.0, and 10.25 GHz
- A single facet model with 46984 triangles used at all frequencies.
- CFIE is used on all interfaces.
- ACA results compared against measurements in UT Austin CEM Benchmark[2]

Frequency (GHz)	N_T	N	M_Z (GB)	C_Z (%)	M_{LU} (GB)	C_{LU} (%)
10.25	46984	70476	2.54	93.1	2.62	93

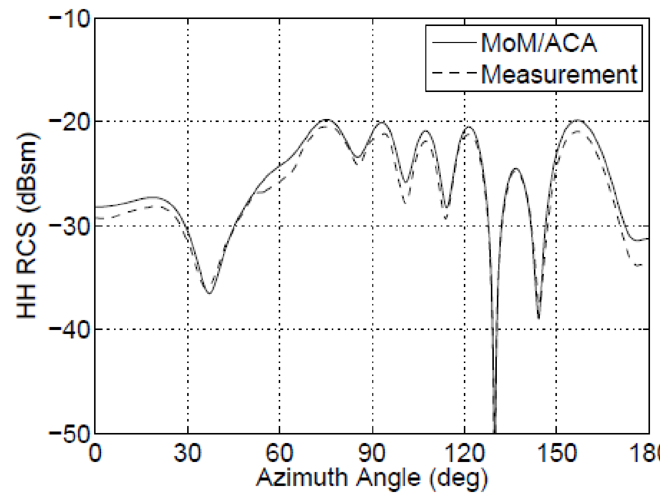
Number of Triangles (N_T), Unknowns (N), MoM Matrix Storage (M_Z) and Percent Compression (C_Z), LU Matrix Storage (M_{LU}) and Percent Compression (C_{LU}),

1. A. C. Woo, H. T. G. Wang, M. J. Schuh, and M. L. Sanders, “Benchmark radar targets for the validation of computational electromagnetics programs,” *IEEE Antennas Propagat. Magazine*, vol. 35, pp. 84–89, February 1993.
2. J. T. Kelley, D. A. Chamulak, C. C. Courtney, and A. E. Yilmaz, “Rye canyon radar cross-section measurements of benchmark almond targets,” *IEEE Antennas Propagat. Mag.*, vol. 52, pp. 120–135, February 2020.

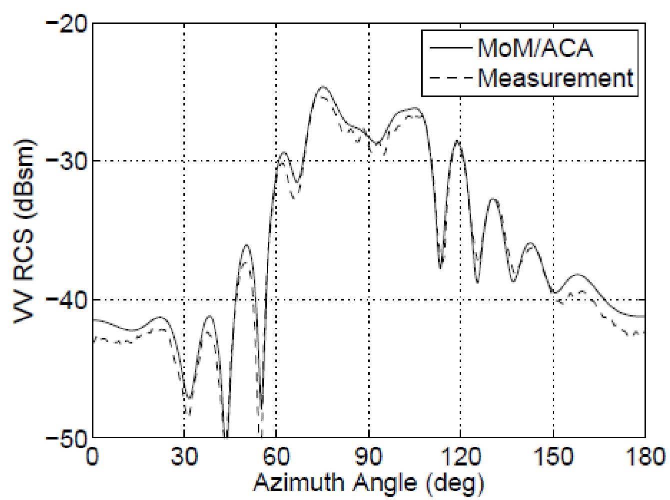
Example: UT Austin PEC Almond (2/3)



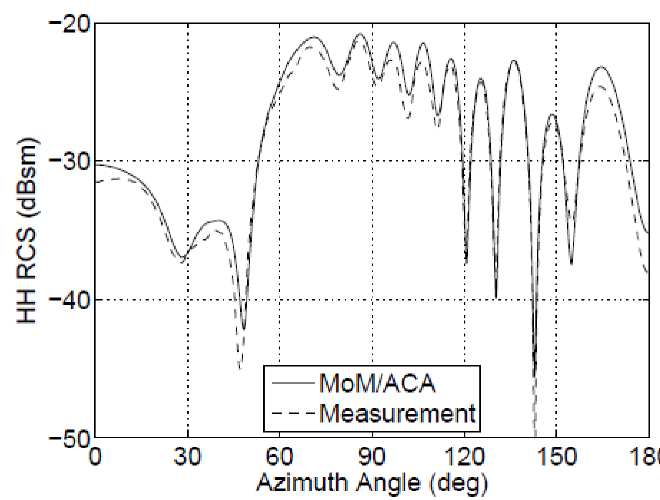
(a) 3.5 GHz, VV-Polarization



(b) 3.5 GHz, HH-Polarization

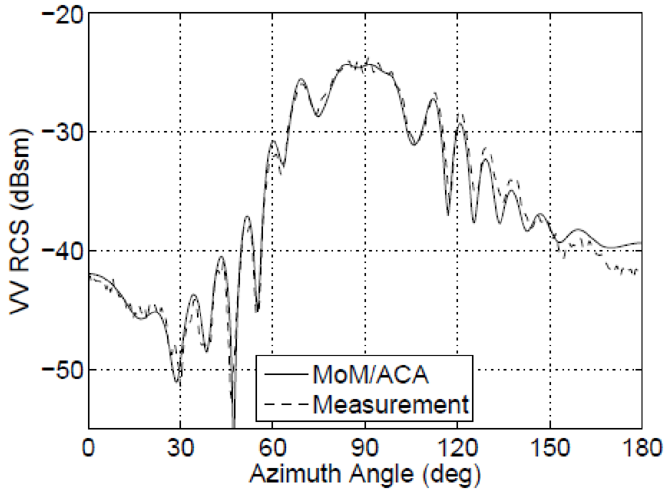


(c) 5.125 GHz, VV-Polarization

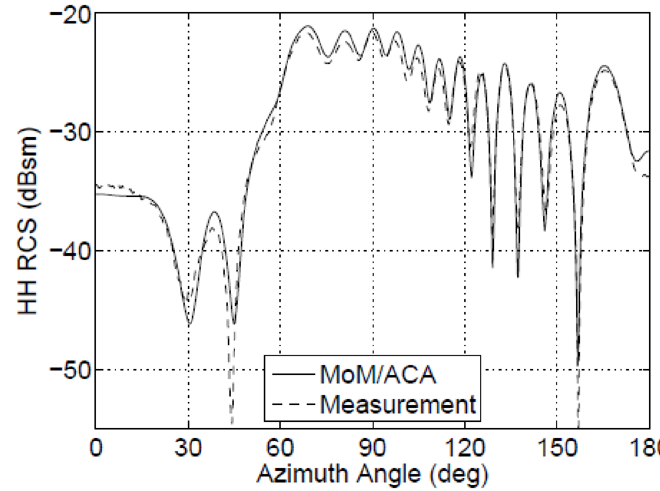


(d) 5.125 GHz, HH-Polarization

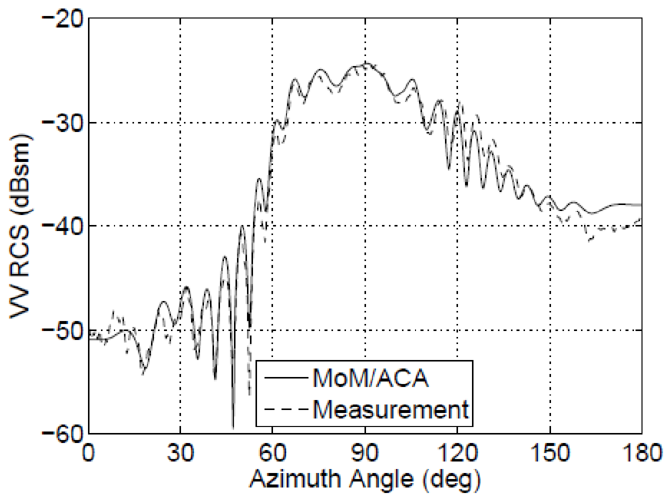
Example: UT Austin PEC Almond (3/3)



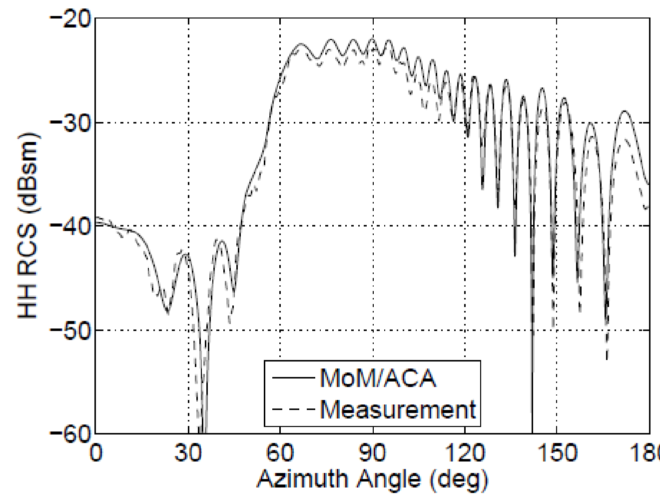
(e) 7.0 GHz, VV-Polarization



(f) 7.0 GHz, HH-Polarization



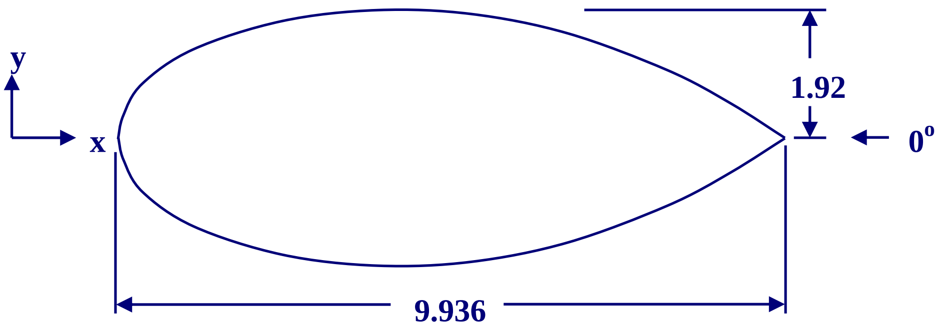
(g) 10.25 GHz, VV-Polarization

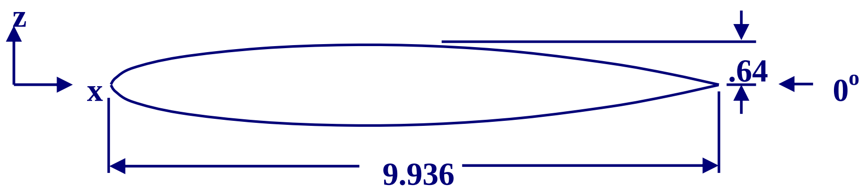


(h) 10.25 GHz, HH-Polarization

Example: UT Austin Solid Dielectric Resin Almond (1/3)

Tripoint Industries, Inc.





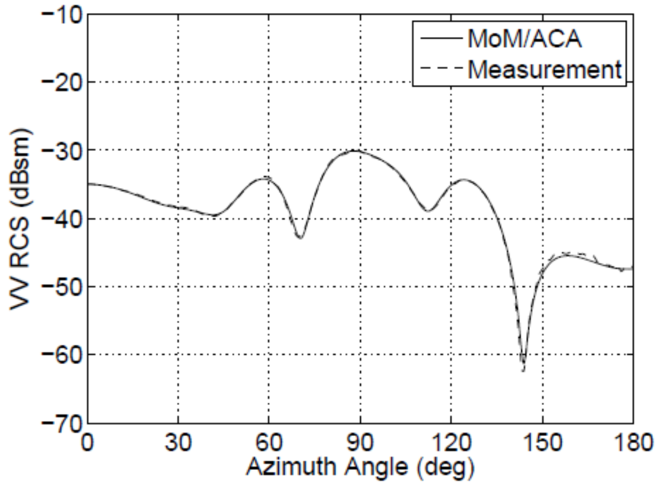
- **Exterior dimensions identical to NASA Almond from the EMCC benchmarks[1]**
- ***Serenity* uses a single-level ACA**
  - **Target ACA group size: 2500**
- **UT Austin test article 3D printed from resin having permittivity e ~ 3 – j.1**
- **RCS computed and compared at 2.58, 5.125, 7.0, and 10.25 GHz**
- **A single facet model with 46984 triangles used at all frequencies.**
- **PMCHWT is used on all interfaces.**
- **ACA results compared against measurements in UT Austin CEM Benchmark[2]**

Frequency (GHz)	N_T	N	M_Z (GB)	C_Z (%)	M_{LU} (GB)	C_{LU} (%)
10.25	46984	140952	8.53	94.2	9.44	93.6

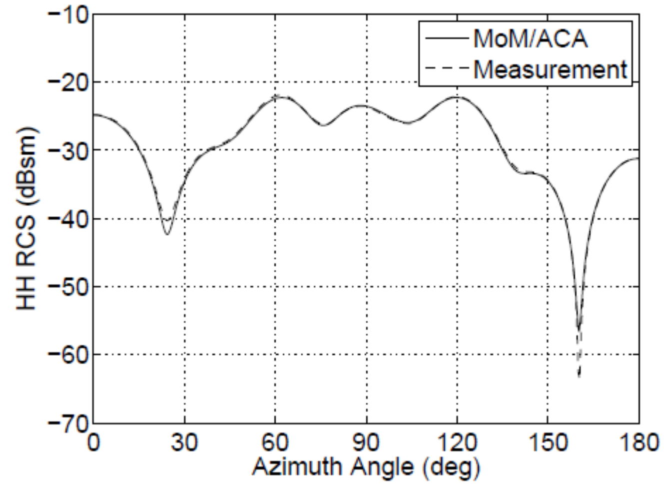
Number of Triangles (N_T), Unknowns (N), MoM Matrix Storage (M_Z) and Percent Compression (C_Z), LU Matrix Storage (M_{LU}) and Percent Compression (C_{LU}),

1. A. C. Woo, H. T. G. Wang, M. J. Schuh, and M. L. Sanders, “Benchmark radar targets for the validation of computational electromagnetics programs,” *IEEE Antennas Propagat. Magazine*, vol. 35, pp. 84–89, February 1993.
2. J. T. Kelley, D. A. Chamulak, C. C. Courtney, and A. E. Yilmaz, “Measurements of non-metallic targets for the Austin RCS benchmark suite,” in *Proc. Ant. Meas. Tech. Assoc. (AMTA) Symp*, 2019.

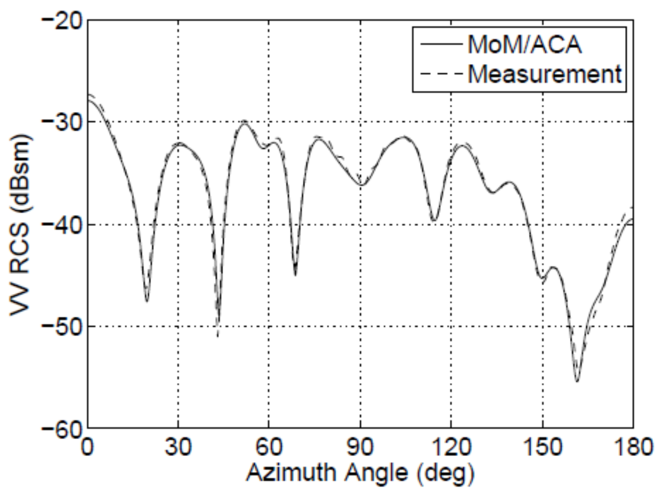
Example: UT Austin Solid Dielectric Resin Almond (2/3)



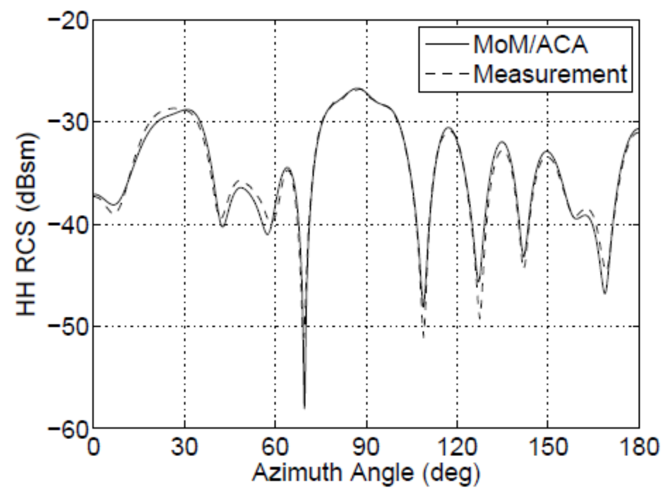
(a) 2.58 GHz, VV-Polarization



(b) 2.58 GHz, HH-Polarization



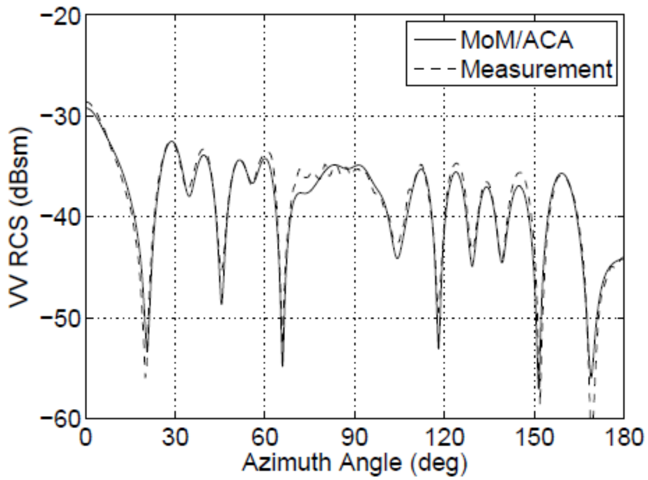
(c) 5.125 GHz, VV-Polarization



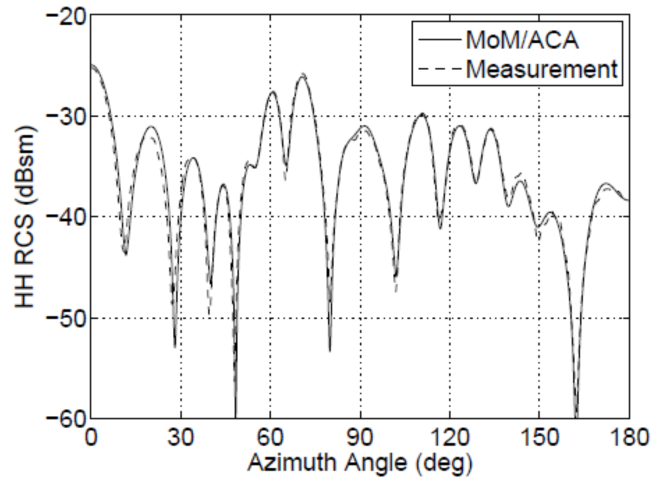
(d) 5.125 GHz, HH-Polarization

Example: UT Austin Solid Dielectric Resin Almond (3/3)

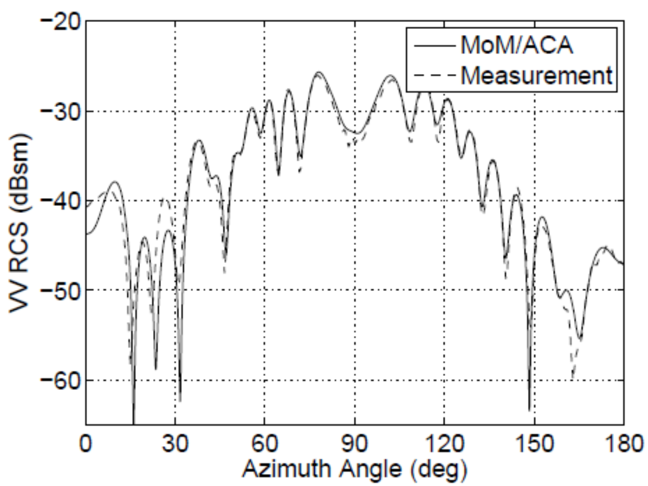
Tripoint Industries, Inc.



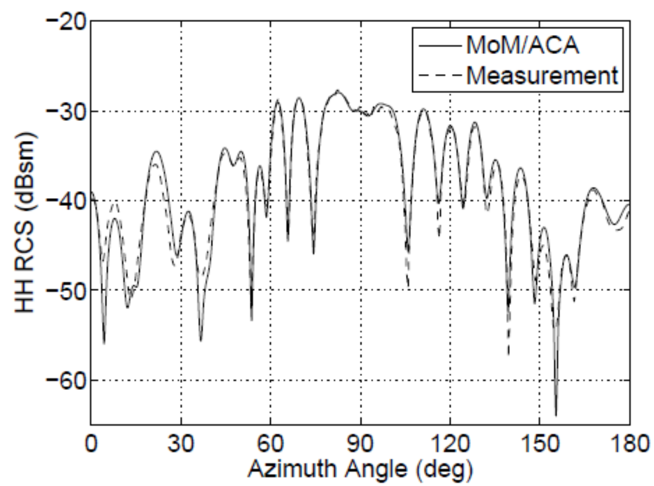
(e) 7.0 GHz, VV-Polarization



(f) 7.0 GHz, HH-Polarization

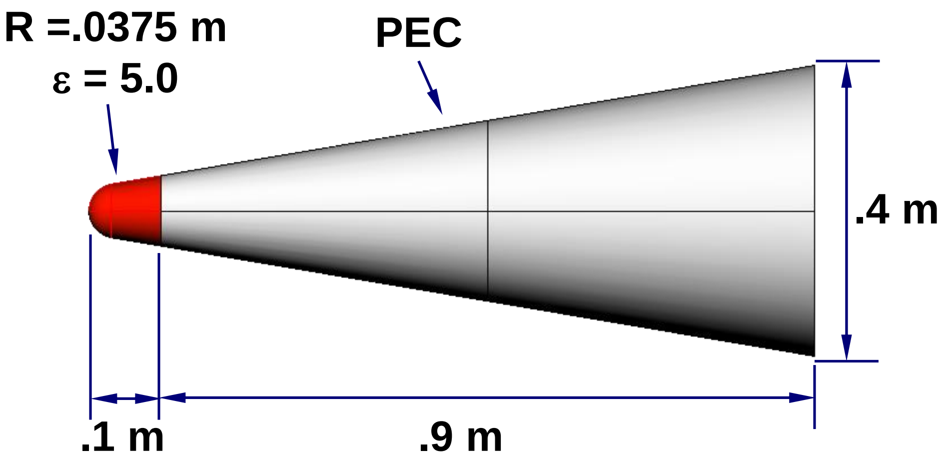


(g) 10.25 GHz, VV-Polarization



(h) 10.25 GHz, HH-Polarization

Example: Reentry Vehicle With Dielectric Nose



- RCS computed at 12, 14, 16, 20 and 24 GHz
- *Serenity* uses a 5-Level MLACA
  - Target ACA group size: 10000
- MLACA results compared against *Galaxy* Body-of-Revolution MoM Solver

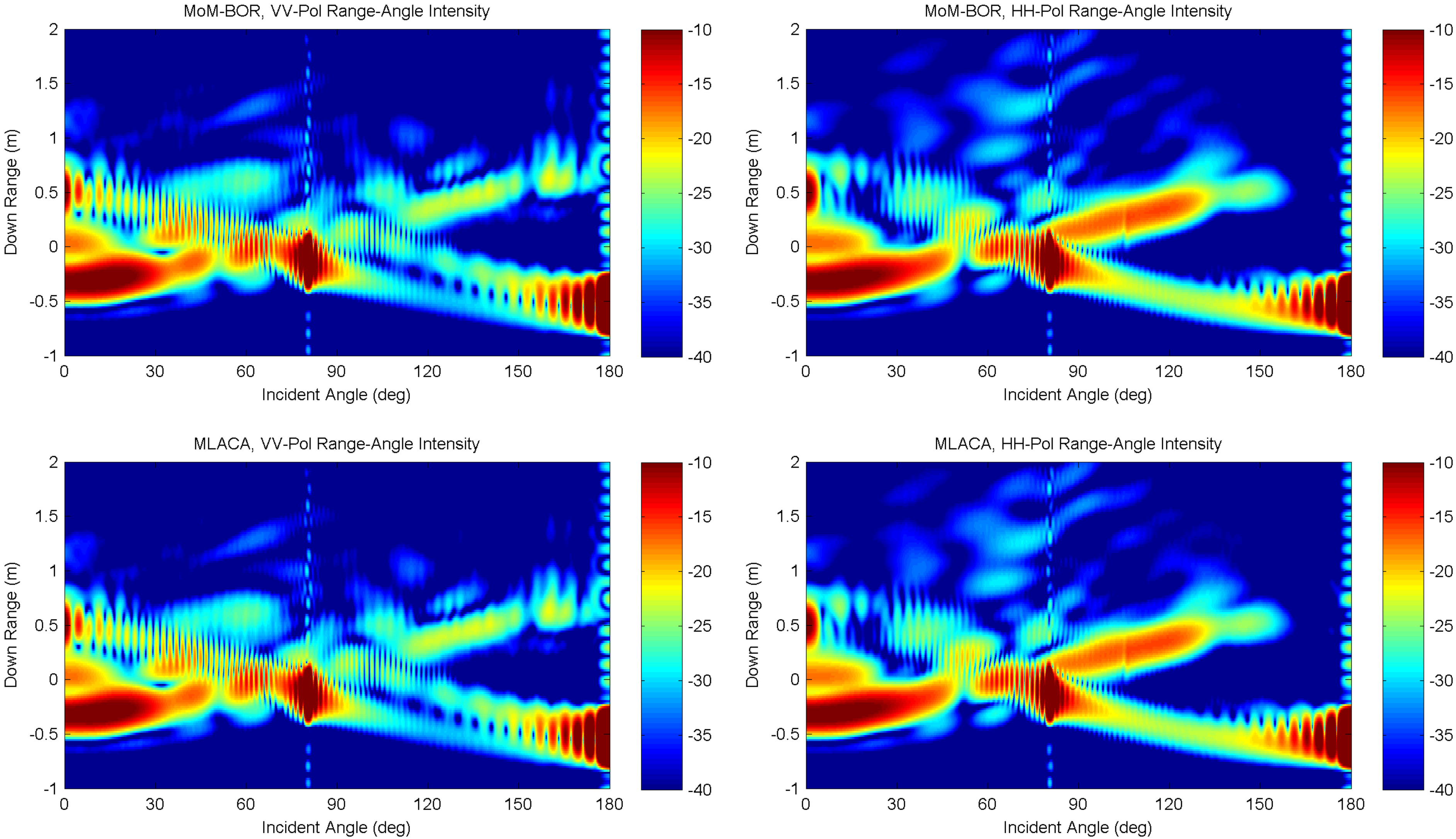
Frequency (GHz)	N_T	N	M_Z (GB)	C_Z (%)	M_{LU} (GB)	C_{LU} (%)
5 - 6	199780	307952	~5		~6	
12	753780	1213896	34	99.69	49	99.55
14	1017940	1645768	52	99.74	80	99.61
16	1086164	1799000	61	99.74	101	99.58
20	1702392	2843288	125	99.79	234	99.61
24	3110440	5022318	291	99.85	554	99.70

Number of Triangles (N_T), Unknowns (N), MoM Matrix Storage (M_Z) and Percent Compression (C_Z), LU Matrix Storage (M_{LU}) and Percent Compression (C_{LU}),

Example: Reentry Vehicle With Dielectric Nose (2)

Tripoint Industries, Inc.

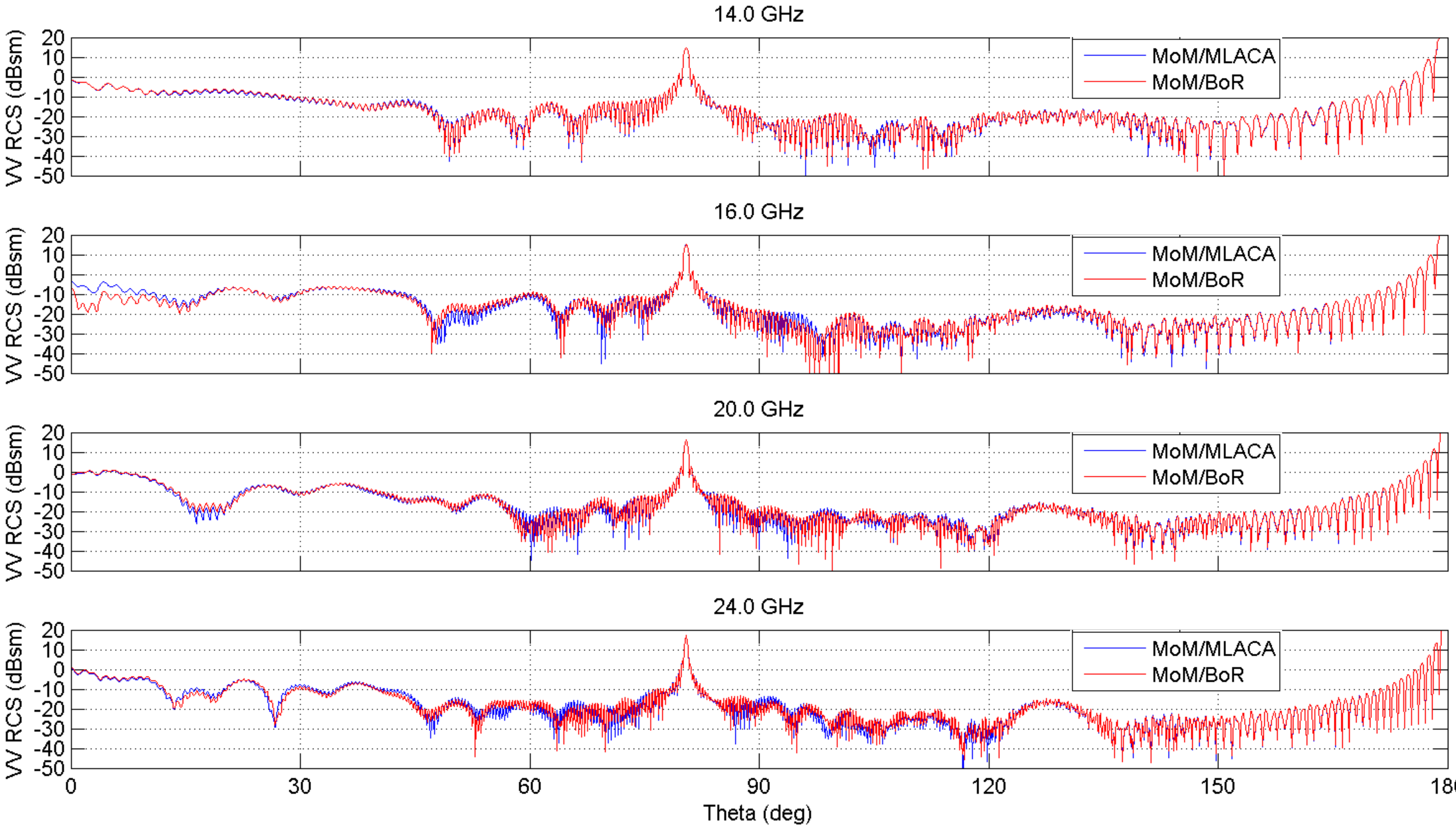
Range-Angle Intensity (RAI) using 5-6 GHz data, results are nearly indistingushable in both polarizations.



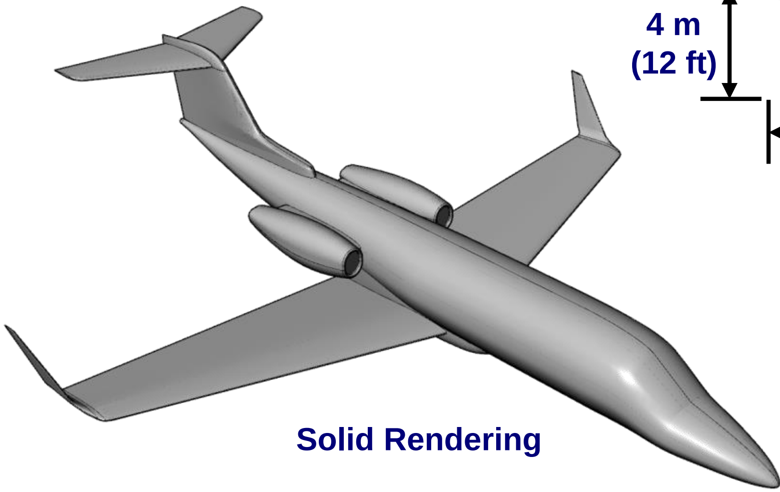
Example: Reentry Vehicle With Dielectric Nose (3)

Tripoint Industries, Inc.

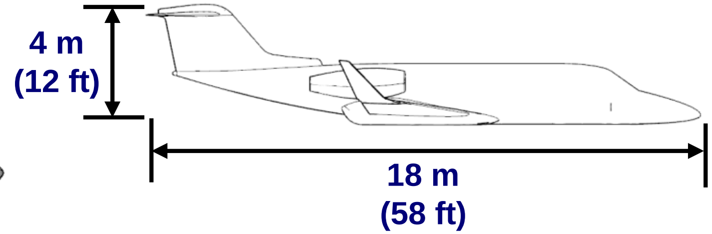
Comparison of VV-Pol RCS at 14, 16, 20 and 24.0 GHz.

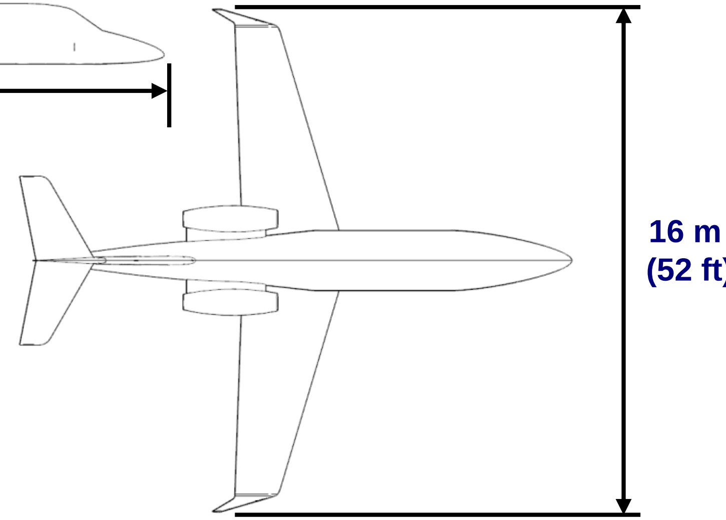


Example: Business Jet



Solid Rendering





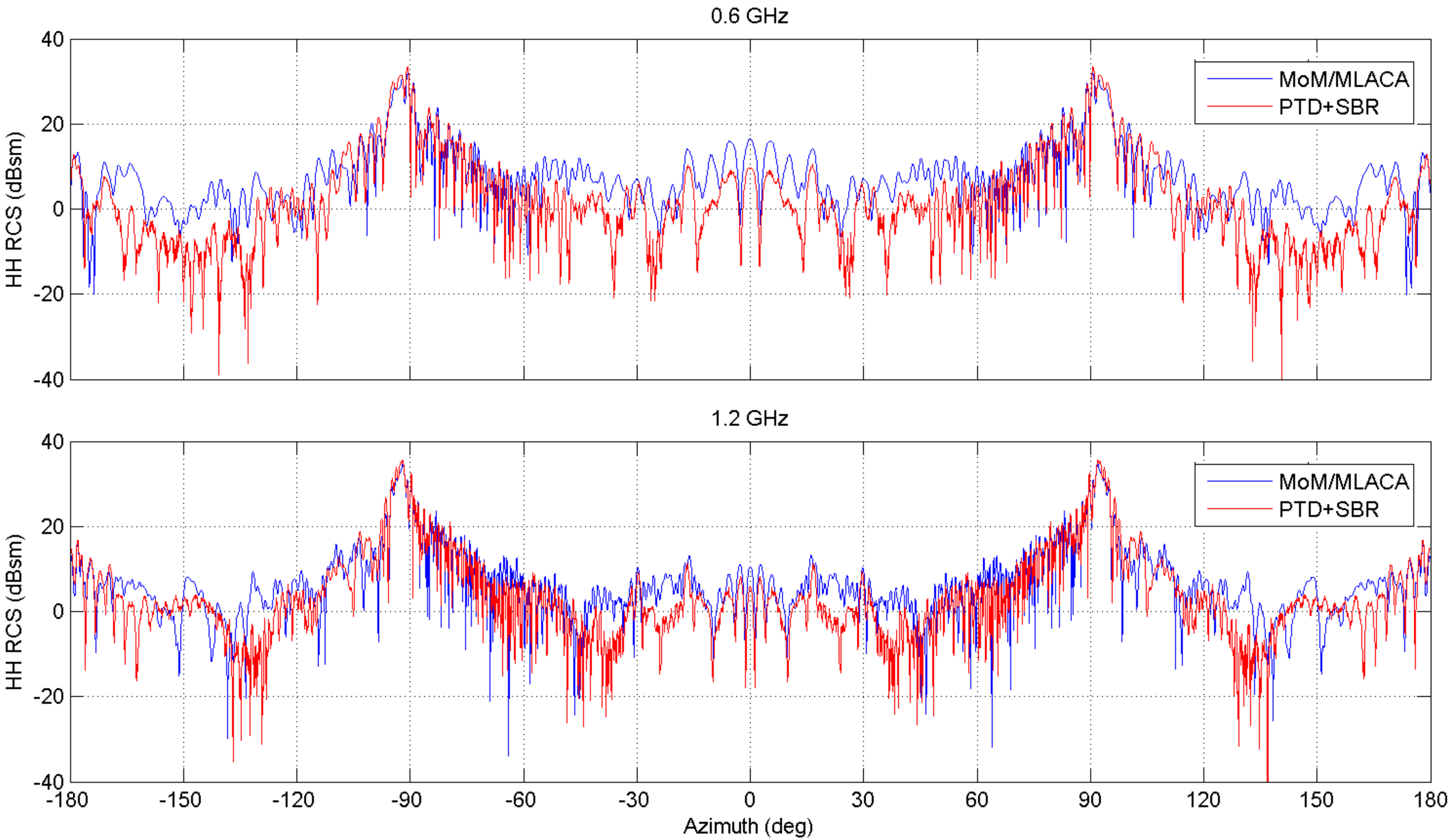
- RCS computed at 0.6, 1.2, 1.8 and 2.4 GHz
- *Serenity* uses a 4-Level MLACA
  - Target ACA group size: 10000
- MLACA results compared against *lucernhammer MT,* our high-frequency PTD/SBR-based Solver

Frequency (GHz)	N_T	N	M_Z (GB)	C_Z (%)	M_{LU} (GB)	C_{LU} (%)
0.6	430392	645588	18	99.43	22	99.28
1.2	1606124	2409186	108	99.75	166	99.61
1.8	3555630	5333445	365	99.83	673	99.68
2.4	6039472	9059208	872	99.86	1746	99.71

Number of Triangles (N_T), Unknowns (N), MoM Matrix Storage (M_Z) and Percent Compression (C_Z),
LU Matrix Storage (M_{LU}) and Percent Compression (C_{LU}),

Example: Business Jet (2)

Comparison of HH-Pol RCS at 0.6 and 1.2 GHz. SBR consistently under-predicts the RCS, as expected.



Example: Business Jet (3)

Tripoint Industries, Inc.

Comparison of HH-Pol RCS at 1.8 and 2.4 GHz. SBR again under-predicts the RCS at many angles.